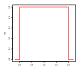
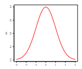
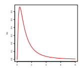
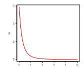
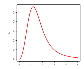
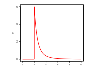
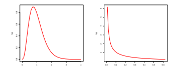
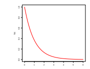
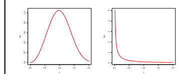
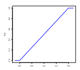
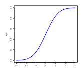
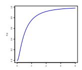
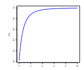
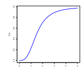
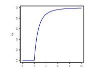
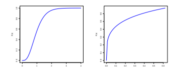
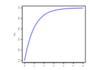
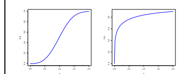
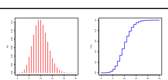
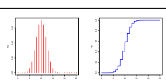
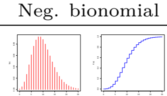
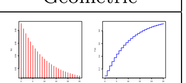
Distribution	Uniform	Normal	Lognormal	Pareto	Loglogistic	Pareto (1p)	Gamma	Exponential	Weibull
$f(x)$	$\frac{1}{b-a}\ \ x \in [a,b]$ 	$\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ 	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-z^2/2} = \frac{\phi(z)}{\sigma x}$ 	$\frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}}$ 	$\frac{\gamma(x/\theta)^\gamma}{x(1+(x/\theta)^2}$ 	$\frac{\alpha\theta^\alpha}{x^{\alpha+1}}\ \ x > \theta$ 	$\frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)}$ 	$\frac{e^{-x/\theta}}{\theta}$ 	$\frac{\tau}{x}\left(\frac{x}{\theta}\right)^\tau e^{-(x/\theta)^\tau}$ 
$F(x)$	$\frac{x-a}{b-a}\ \ x \in [a,b]$ 	$\Phi(x)$ 	$\Phi(z)$ 	$1-\left(\frac{\theta}{x+\theta}\right)^\alpha$ 	$\frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma}$ 	$1-\left(\frac{\theta}{x}\right)^\alpha$ 	$\Gamma(\alpha; x/\theta)$ 	$1-e^{-x/\theta}$ 	$1-e^{-(x/\theta)^\tau}$ 
$h(x) = \frac{f(x)}{S(x)} = \frac{f(x)}{1-F(x)}$	$\frac{1}{b-x}$			$\frac{\alpha}{x+\theta}$		$\frac{\alpha}{x}$		$\frac{1}{\theta}$	$\frac{\tau}{x}\left(\frac{x}{\theta}\right)^\tau = \frac{\tau}{\theta^\tau}x^{\tau-1}$
$E[X] = \mu$	$\frac{a+b}{2}$	μ	$e^{\mu+\sigma^2/2}$	$\frac{\theta}{\alpha-1}$	$\theta\Gamma(1+\frac{1}{\gamma})\Gamma(1-\frac{1}{\gamma})$	$\frac{\alpha\theta}{\alpha-1}$	$\theta\alpha$	θ	$\theta\Gamma(1+\frac{1}{\tau})$
$E[X^2] = \mu'_2$	$\frac{b^3-a^3}{3(b-a)}$	$\mu^2+\sigma^2$	$e^{2\mu+2\sigma^2}$	$\frac{2\theta^2}{(\alpha-1)(\alpha-2)}$	$\theta^2\Gamma(1+\frac{2}{\gamma})\Gamma(1-\frac{2}{\gamma})$	$\frac{\alpha\theta^2}{\alpha-2}$	$\alpha(\alpha+1)\theta^2$	$2\theta^2$	$\theta^2\Gamma(1+\frac{2}{\tau})$
$\sigma^2 = \mu'_2 - \mu^2$	$\frac{(b-a)^2}{12}$	σ^2	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$				$\alpha\theta^2$	θ^2	
$\sigma = \sqrt{\sigma^2}$	$\frac{(b-a)}{\sqrt{12}}$	σ	$e^{\mu+\sigma^2/2}\sqrt{e^{\sigma^2}-1}$				$\sqrt{\alpha}\theta$	θ	
$E[X^3] = \mu'_3$	$\frac{b^4-a^4}{4(b-a)}$			$\frac{18\theta^3}{(\alpha-1)(\alpha-2)(\alpha-3)}$	$\theta^3\Gamma(1+\frac{3}{\gamma})\Gamma(1-\frac{3}{\gamma})$	$\frac{\alpha\theta^3}{\alpha-3}$	$\alpha(\alpha+1)(\alpha+2)\theta^3$	$6\theta^3$	$\theta^3\Gamma(1+\frac{3}{\tau})$
$E[X^4] = \mu'_4$	$\frac{b^5-a^5}{5(b-a)}$			$\frac{4!\theta^3}{(\alpha-1)\dots(\alpha-4)}$	$\theta^4\Gamma(1+\frac{4}{\gamma})\Gamma(1-\frac{4}{\gamma})$	$\frac{\alpha\theta^4}{\alpha-4}$	$\alpha(\alpha+1)(\alpha+2)(\alpha+3)\theta^4$	$24\theta^4$	$\theta^4\Gamma(1+\frac{4}{\tau})$
$E[X^k] = \mu'_k$	$\frac{b^{k+1}-a^{k+1}}{(k+1)(b-a)}$		$e^{k\mu+k^2\sigma^2/2}$	$\frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}\ \ k \in \mathbb{N}$	$\theta^k\Gamma(1+\frac{k}{\gamma})\Gamma(1-\frac{k}{\gamma})$	$\frac{\alpha\theta^k}{\alpha-k}\ \ k < \alpha$	$\theta^k(\alpha+k-1)\cdots\alpha$	$\theta^k k!\ \ k \in \mathbb{N}$ (2)	$\theta^k\Gamma(1+\frac{k}{\tau})$
$E[(X-\mu)^3] = \mu_3$	0	0							
$E[(X-\mu)^4] = \mu_4$									
CoV: $\frac{\sigma}{\mu}$	$\frac{1}{6}$	$\frac{\sigma}{\mu}$	$\sqrt{e^{\sigma^2}-1}$				$\frac{\sqrt{\alpha\theta^2}}{\alpha\theta} = \alpha^{-1/2}$	$\frac{\theta}{\theta} = 1$	
Skewness: $\gamma_1 = \frac{\mu_3}{\sigma^3}$	0	0							
Kurtosis: $\gamma_2 = \frac{\mu_4}{\sigma^4}$		3							
Mode		μ	$e^{\mu-\sigma^2}$	0	$\theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}$	θ	$\theta(\alpha-1)\ \alpha > 1$ else 0	0	$\theta\left(\frac{\tau-1}{\tau}\right)^{1/\tau}$
Median		0	e^μ			$\theta(2)^{1/\alpha}$		$-\theta\ln\frac{1}{2} = \theta\ln 2$	
$F^{-1}(u)\ \ u = U(0,1)$	$u(b-a)+a$	$\mu+\Phi^{-1}(u)\sigma$	$e^{\mu+\Phi^{-1}(u)\sigma}$	$\theta((1-u)^{-\frac{1}{\alpha}}-1)$	$\theta\left(\frac{u}{1-u}\right)^{1/\gamma}$	$\theta(1-u)^{-1/\alpha}$		$-\theta\ln(1-u)$	
$e_x^k(d) = \frac{\int_d^\infty (x-d)^k f(x)dx}{1-F(d)}$	$\frac{w-d}{2}$			$\frac{d+\theta}{\alpha-1}$				θ (memoryless)	
MGF: $M_x(t) = E[e^{tX}]$	$\frac{e^{bt}-e^{at}}{(b-a)t}$	$e^{\mu t+\frac{\sigma^2t^2}{2}}$		No MGF			$\left(\frac{1}{1-\theta t}\right)^\alpha\ \ t < \frac{1}{\theta}$	$\frac{1}{1-\theta t}$	
PGF: $P_x(z) = E[z^x]$									
VaR_p		$\mu+\sigma\Phi^{-1}(p)$		$\theta((1-p)^{-1/\alpha}-1)$	$\theta(p^{-1}-1)^{-1/\gamma}$			$-\theta\ln(1-p)$	
$\mathrm{TVaR}_p = \mathrm{VaR}_p + e(\mathrm{VaR}_p)$		$\mu+\sigma\frac{\phi(\Phi^{-1}(p))}{1-p}$		$\mathrm{VaR}_p + \frac{\theta(1-p)^{-1/\alpha}}{\alpha-1}$				$-\theta\ln(1-p)+\theta$	
Limited e.v. $E[X \wedge x]$			$e^{\mu+\sigma^2/2}$	$\frac{\theta}{\alpha-1}\left(1-\left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right)$ (4)				$\theta(1-e^{-x/\theta})$	
$E[(X \wedge x)^k]$			(1)					(3)	
Notes			$z = \frac{\ln x-\mu}{\sigma}$				L: $\alpha > 1$, R: $\alpha < 1$	Gamma $\alpha = 1$	L: $\tau > 1$, R: $\tau < 1$
Parameter role		Is scale distrib no scale param.	μ: location (no scale) θ: shape	α: shape θ: shape			μ: shape		
R sim. function	runif(n,a,b)	rnorm(n,μ,σ)	rlnorm(n,μ,σ)	rpareto(n,α,θ)	rllogis(n,γ,θ)	rpareto1(n,α,θ)	rgamma(n,,)	rweibull(n,α,θ)	rexp(n,θ)

Distribution	Poisson	Binomial	Neg. bionomial	Geometric
p_0	 $e^{-\lambda}$	 $(1-q)^m$	 $(1+\beta)^{-r} = (\frac{1}{1+\beta})^r$	 $(1+\beta)^{-1} = \frac{1}{1+\beta}$
$p_k = (a+\frac{b}{k})p_{k-1}$	$\frac{e^{-\lambda}\lambda^k}{k!}$	$\binom{m}{k}q^k(1-q)^{m-k}$	$\binom{k+r-1}{k}(\frac{1}{1+\beta})^r(\frac{\beta}{1+\beta})^k$	$(\frac{1}{1+\beta})(\frac{\beta}{1+\beta})^k$
a	0	$-\frac{q}{1-q} < 0$	$\frac{\beta}{1+\beta} > 0$	$\frac{\beta}{1+\beta}$
b	λ	$(m+1)\frac{q}{1-q}$	$(r-1)\frac{\beta}{1+\beta}$	0 (memoryless)
E[N]	λ	mq	$r\beta$	β
Var[N]	λ	$mq(1-q)$	$r\beta(1+\beta)$	$\beta(1+\beta)$
$P_X(z) = M_X(\ln z)$	$e^{\lambda(z-1)}$	$(1+q(z-1))^m$	$(1-\beta(z-1))^{-r}$	$(1-\beta(z-1))^{-1}$
$M_X(t) = P_X(e^t)$	$\exp(\lambda(e^t-1))$			
Note		$0 < q < 1,\ m \in \mathbb{N}$	$r > 0\ \beta > 0$	NegBin $r = 1$
R sim. function	rpois(n,λ)	rbinom(n,m,q)	rnbinom(n,r,$\frac{1}{1+\beta}$)	rgeom(n,$\frac{1}{1+\beta}$)

(1) $e^{k\mu+k^2\sigma^2/2}\Phi\left(\frac{\ln x-\mu-k\sigma^2}{\theta}\right) + x^k(1-F(x))$ (2) Special case for k > -1 (3) See Table. (4) Special case for $\alpha = 1$

$$\Gamma(\alpha; x) = \frac{1}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-t}dt \quad \alpha > 0, x > 0 \quad \text{with} \quad \Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}dt$$

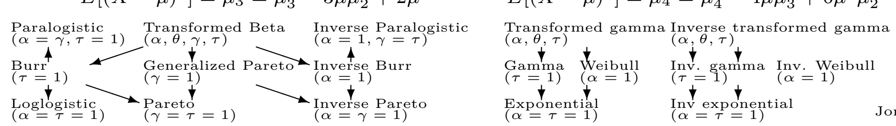
$$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1) \qquad \Gamma(n) = (n-1)! \text{ for } n \in \mathbb{N}$$

$$\binom{x}{k} = \frac{x(x-1)\cdots(x-k+1)}{k!} \quad k \in \mathbb{N}, x \in \mathbb{R} \qquad \binom{x}{k} = \frac{\Gamma(x+1)}{\Gamma(k+1)\Gamma(x-k+1)} = \frac{x!}{k!(x-k)!} \quad k, x \in \mathbb{N}$$

The **kth central moment** $E[(x-\mu)^k] = \mu_k$ • **Variance**: $\mu_2 = \sigma^2$ • **Standard deviation**: $\sqrt{\mu_2} = \sigma$ • **Coefficient of variation**: σ/μ • **Skewness**: $\gamma_1 = \mu_3/\sigma^3$ • **Kurtosis**: $\gamma_2 = \mu_4/\sigma^4$

$E[X] = \mu$ $\quad E[X^2] = \mu'_2$ $\quad E(X^3) = \mu'_3,$ $\quad E(X^4) = \mu'_4,$ $\quad E[(X-\mu)^2] = \mu_2 = \sigma^2 = \mu'_2 - \mu^2$

$E[(X-\mu)^3] = \mu_3 = \mu'_3 - 3\mu\mu'_2 + 2\mu^3$ $\quad E[(X-\mu)^4] = \mu_4 = \mu'_4 - 4\mu\mu'_3 + 6\mu^2\mu'_2 - 3\mu^4$



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